5 – Surface Stress Detection

ME-426 – Micro/Nanomechanical Devices

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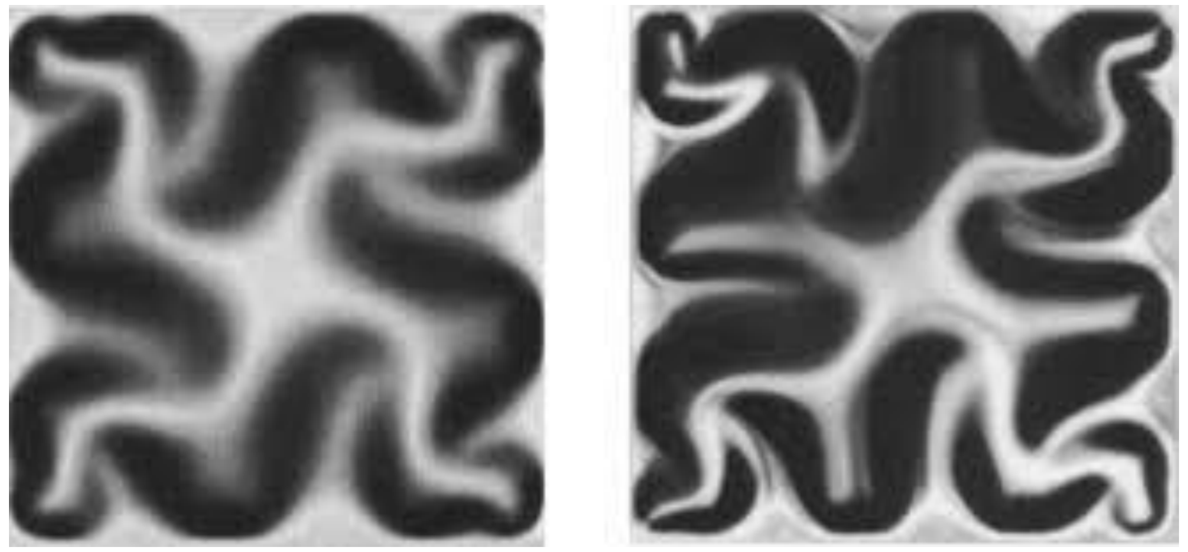
Introduction

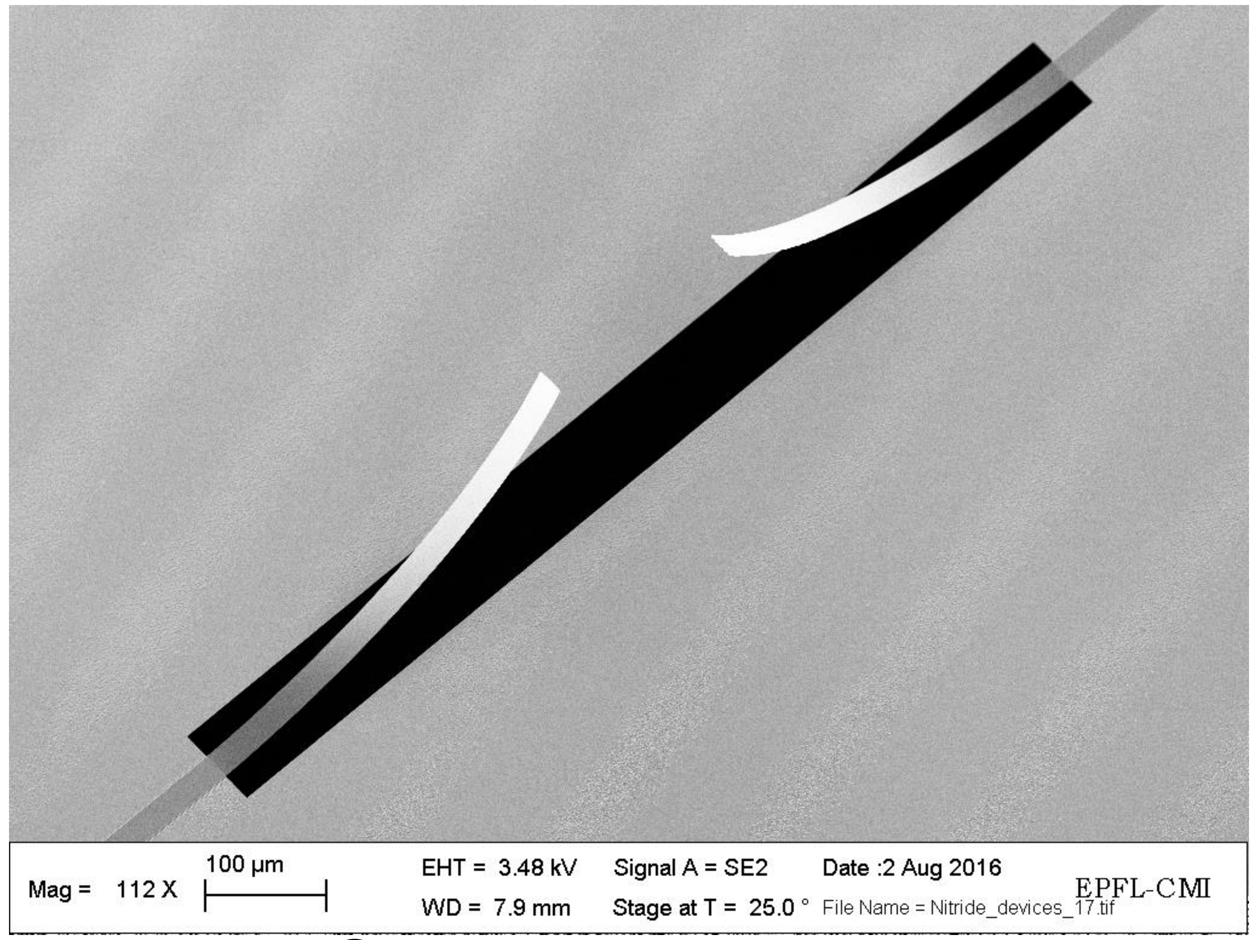
- Transduction chain
- Surface Stress
- Bimorph structure
- Stress gradient

Surface Stress Detection

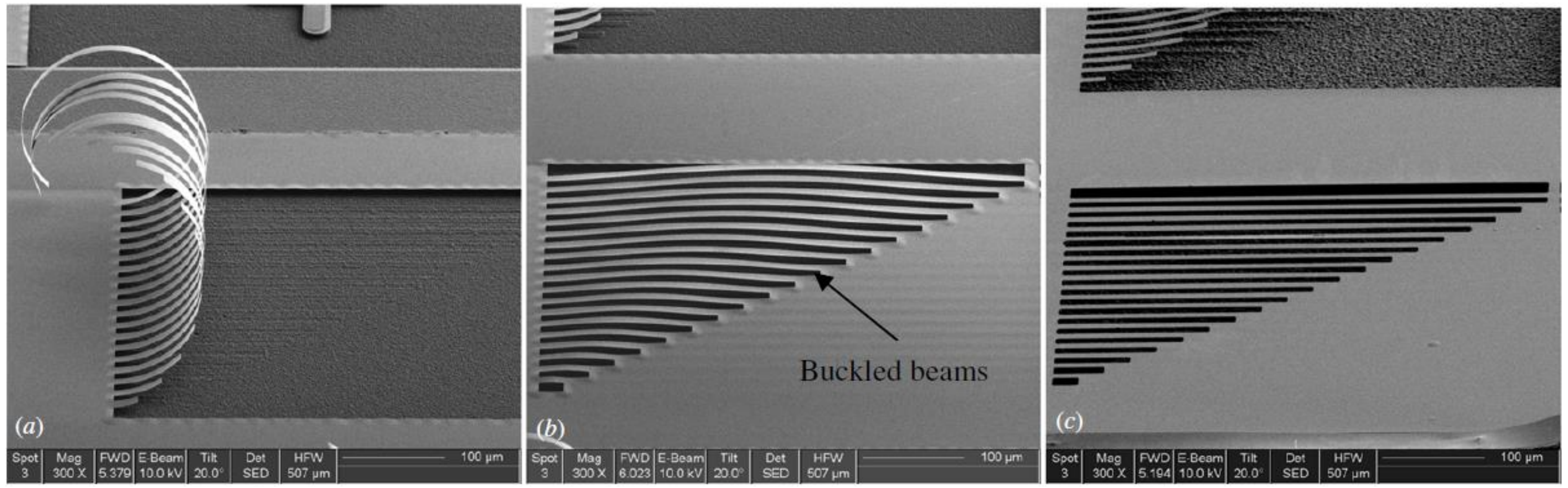
Stress in thin films!!!



Buckling



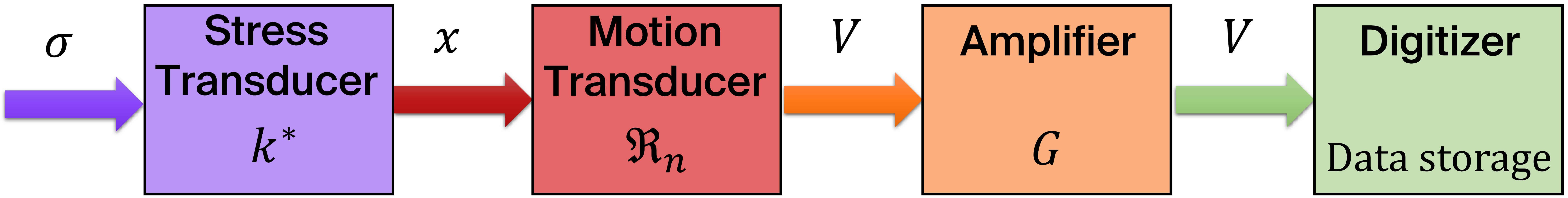
Stress gradient



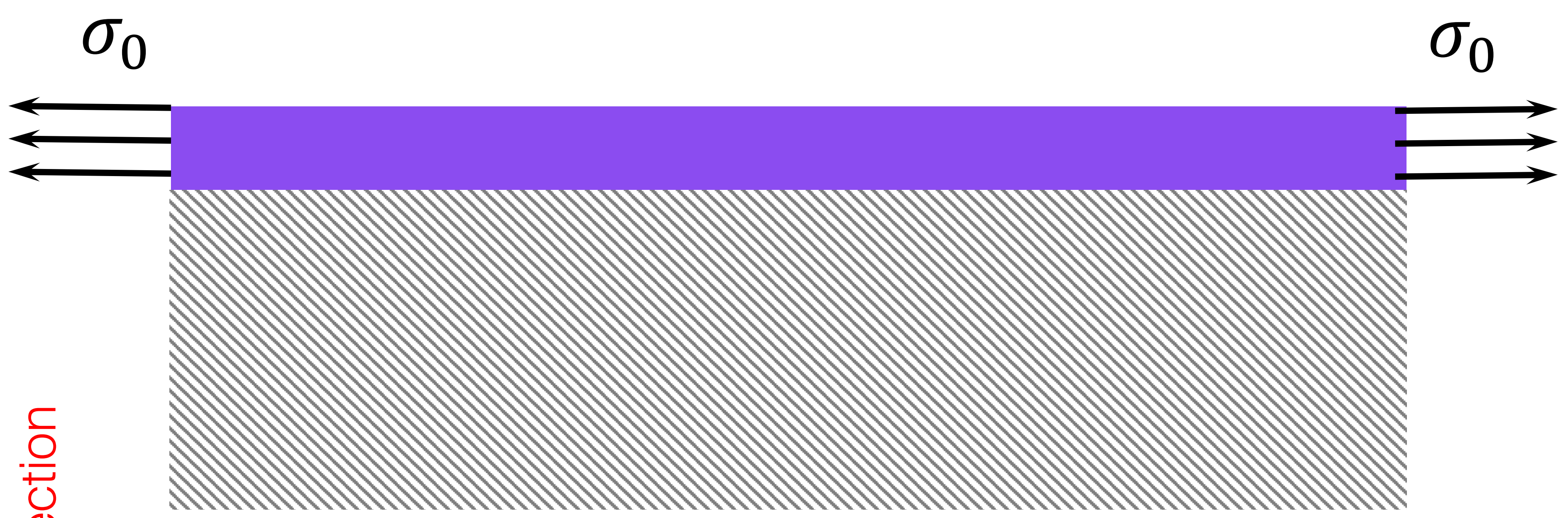
Stress gradient

Buckling

Stress detection



What happens after release?



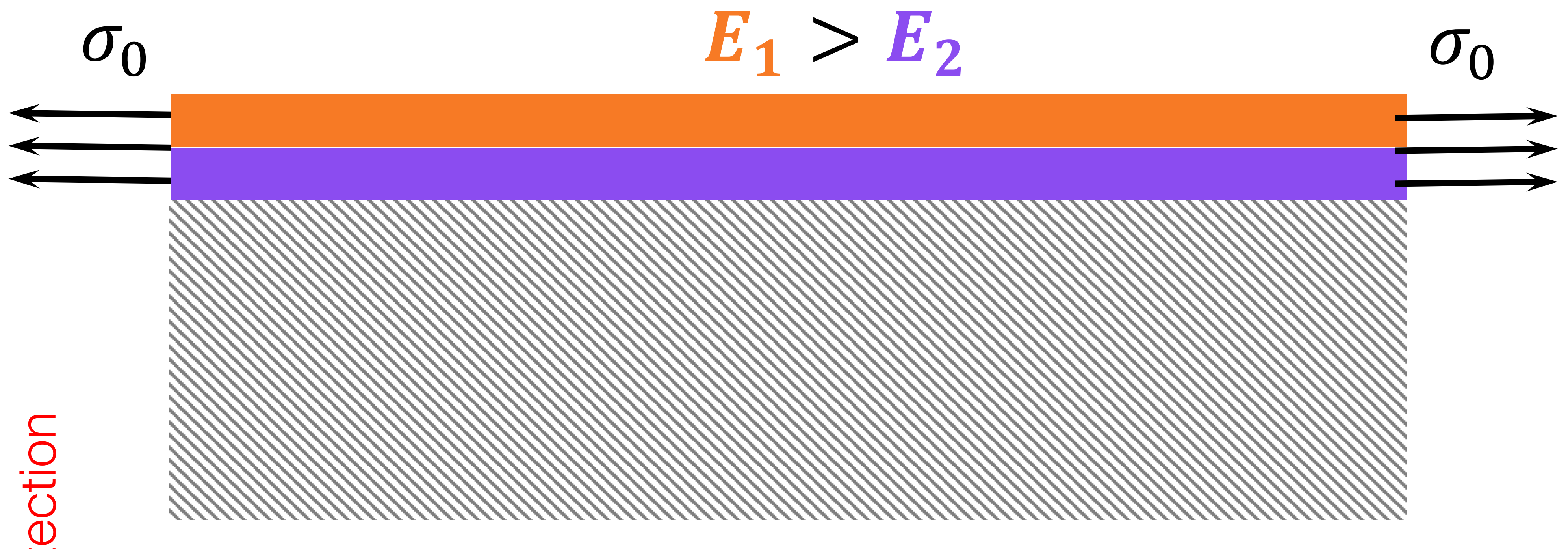
A. Beam shortens

B. Beam bends up

C. Beam bends down

D. Beam does not move

What happens after release?



A. Beam shortens

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Multi-layer structure

- We assume all the time $L \gg w, t$

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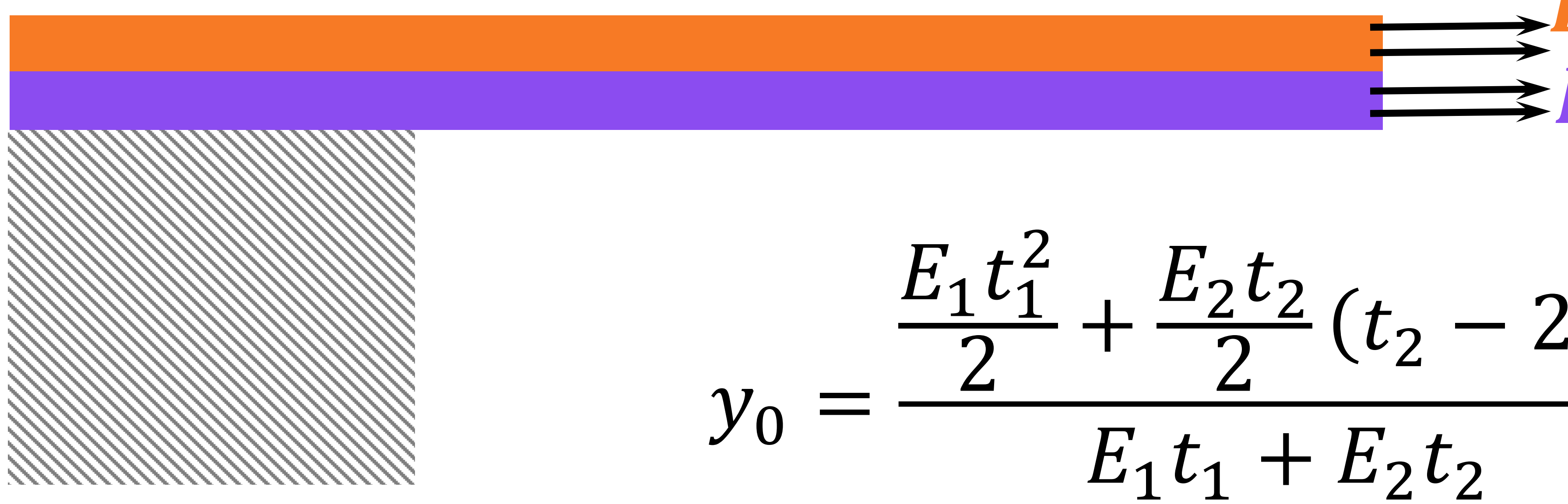


$$y_0 = \frac{\frac{E_1 t_1^2}{2} + \frac{E_2 t_2}{2}(t_2 - 2t_1)}{E_1 t_1 + E_2 t_2}$$

- We assume all the time $L \gg w, t$
- When we release, we have the following internal “loads”:
- $N_{in}(x) = (\sigma_1 t_1 + \sigma_2 t_2)w$
- $M_{in}(x) = -\int_0^{t_{tot}} \sigma(y)(y - y_0)w\,dy = -\int_0^{t_1} \sigma_1 yw\,dy - \int_{t_1}^{t_{tot}} \sigma_2 yw\,dy + y_0 N(x) =$

$$= -\frac{wt_1^2}{2} - \frac{\sigma_2 w t_2^2}{2}(t_2 - 2t_1) + y_0 N(x)$$

Multi-layer structure

- We assume all the time $L \gg w, t$

- Writing: $\varepsilon(y) = \varepsilon_0 - \frac{1}{\rho}(y - y_0)$

$$\varepsilon_0 = -\frac{t_1\sigma_1 + t_2\sigma_2}{t_1E_1 + t_2E_2}$$

$$\frac{1}{\rho} = -\frac{6t_1t_2(t_1 + t_2)(E_2\sigma_1 - E_1\sigma_2)}{t_1^4E_1^2 + 2t_1t_2E_1E_2(2t_1^2 + 3t_1t_2 + 2t_2^2) + t_2^4E_2^2}$$

Multi-layer structure – Case 1

- We assume all the time $L \gg w, t$
- Bottom layer stress-free, top layer very thin

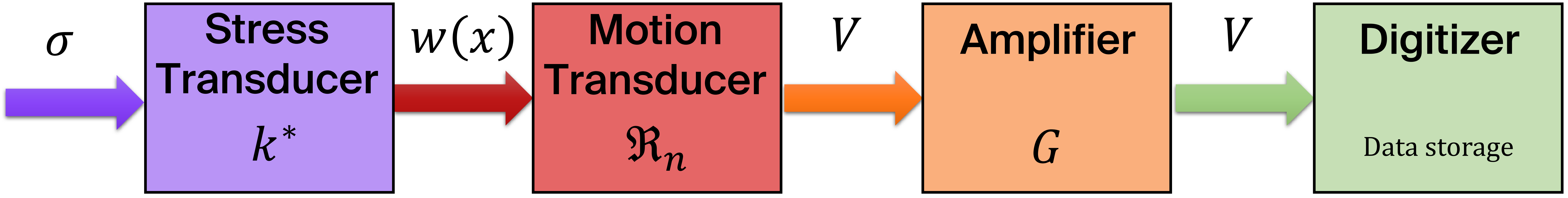
$$\varepsilon_0 = -\frac{t_1\sigma_1 + t_2\sigma_2}{t_1E_1 + t_2E_2} \approx -\frac{t_2\sigma_2}{t_1E_1}$$

$$\frac{1}{\rho} \approx \frac{6t_2\sigma_2}{t_1^2E_1} = -\frac{6\varepsilon_0}{t_1}$$

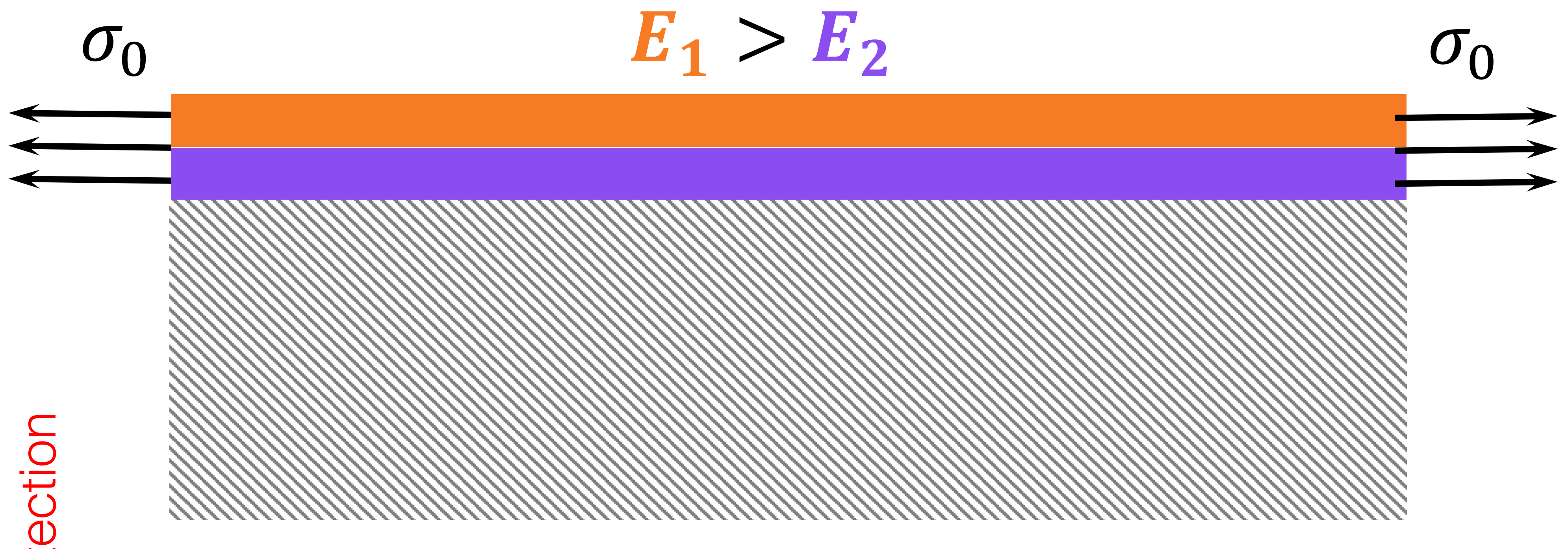
How much is the elastic constant?

$$\frac{1}{\rho} \approx \frac{6t_2\sigma_2}{t_1^2 E_1}$$

$$\frac{\partial^2 w}{\partial x^2} = \frac{1}{\rho} \rightarrow w(x) \approx \frac{6t_2\sigma_2}{t_1^2 E_1}\frac{x^2}{2} = \frac{3t_2\sigma_2}{t_1^2 E_1}x^2$$



What happens after release?



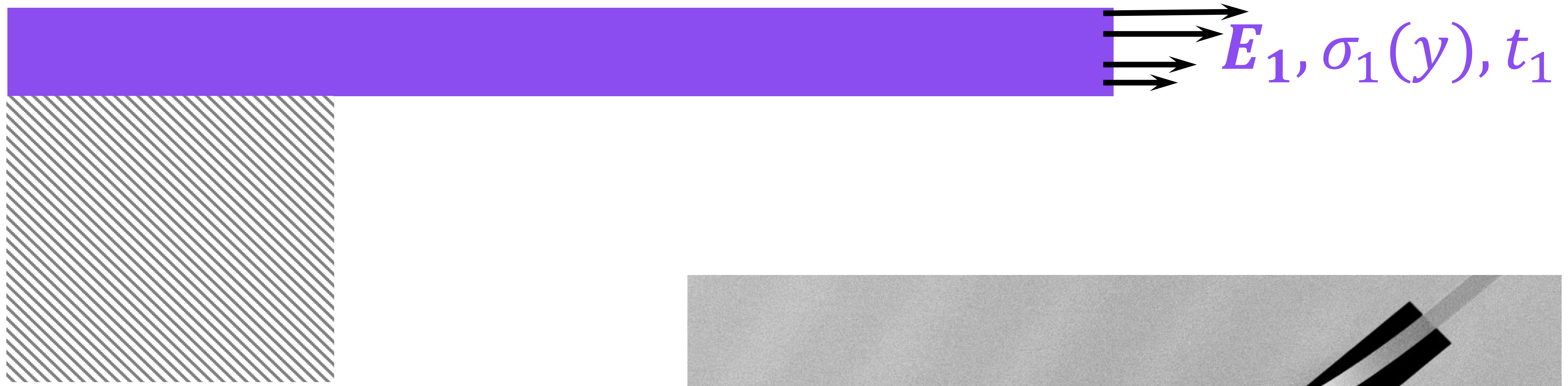
A. Beam elongates

B. Beam bends up

C. Beam bends down

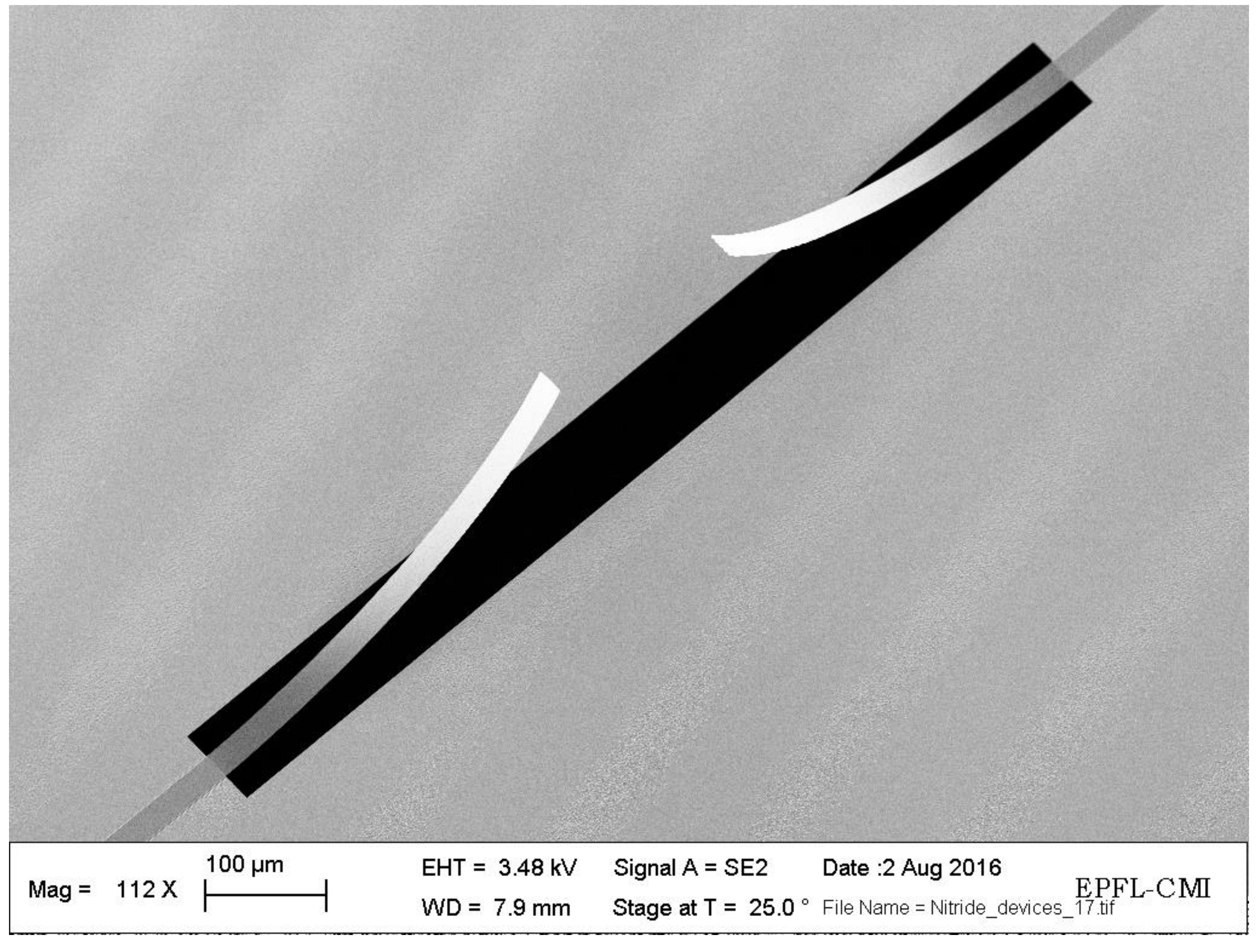
D. Beam does not move

Single layer structure – Stress gradient



- We assume all the time $L \gg w, t$

$$y_0 = \frac{t_1}{2} \qquad \sigma_1(y) = \sigma_{av} + \gamma(y - y_0)$$



- When we release, we have the following internal “loads”:

- $N_0(x) = \sigma_{1,av} t_1 w$

- $M_0(x) = -\int_0^{t_1} \sigma_1(y)(y - y_0) w \, dy = -\int_0^{t_1} \left(\sigma_{av} + \gamma(y - y_0)\right)(y - y_0) w \, dy =$

$$= -\int_0^{t_1} \sigma_{av}(y - y_0) w \, dy - \int_0^{t_1} \gamma (y - y_0)^2 w \, dy = -\gamma I$$

$$\varepsilon(y) = \varepsilon_0 - \frac{1}{\rho}(y - y_0) \rightarrow \varepsilon_0 = -\frac{\sigma_{av}}{E_1}; \qquad \frac{1}{\rho} = \frac{\gamma}{E}$$